

On non-commutative $\mathcal{N} = 2$ super Yang-Mills

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Abstract

We discuss the Seiberg-Witten solution of the non-commutative $\mathcal{N} = 2$ $U(N)$ SYM model. The solution is described in terms of the ordinary Seiberg-Witten curve of the $SU(N)$ theory plus an additional free $U(1)$. Hence, at the two-derivative approximation the theory flows to the ordinary commutative theory in the infra-red ($k < 1/\sqrt{\theta}$). In particular, the center $U(1)$ is free and it decouples from the other $U(1)$'s. In addition, no UV/IR mixing is found.

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1 Introduction

Non-commutative field theories have recently attracted much attention, mainly due to the discovery of their connection to string theory [1, 2]. The perturbative structure of these theories is interesting [3]. Though they differ from ordinary theories by higher derivative terms, perturbative results seem to indicate that they do not flow to the ordinary theory in the infrared. Contributions from the UV affect the IR. This surprising behavior, called UV/IR mixing, seems to contradict the Wilsonian picture. We usually refer to higher dimensional operators as 'irrelevant' in the IR. However, in the case of non-commutative theories, an infinite sum of irrelevant contributions conspire to be relevant in the IR.

The picture just described rests on one-loop calculations [4]. In order to gain a complete understanding of the IR, non-perturbative methods might be required. In the case of $\mathcal{N} = 4$ SYM, it was found, by using the AdS/CFT correspondence, that there is no UV/IR mixing and the theory converges to the commutative one in the IR [5, 6]. The reason is that the non-planar contributions, which are responsible to the UV/IR mixing, cancel in this case [4].

A model for analyzing the infra-red behavior of the non-commutative theories is pure $\mathcal{N} = 2$ SYM. This theory is not finite and à priori we expect UV/IR mixing. Moreover, using the Seiberg-Witten theory[7] we can analyze the IR physics. Several attempts in this directions were already made. In the first [8] the fate of the center of the gauge group was not addressed. We disagree with the results of the two other papers [9, 10].

The one-loop effective action of the NC $U(1)$ $\mathcal{N} = 2$ SYM was recently derived [11, 12]. Based on this one-loop calculation, it was suggested [11] that the $U(1)$ theory flows asymptotically to a free theory in the IR. Note, however, that non-perturbative effects might be essential for a generic value of θ .²

In order to describe the theory at the IR, we parametrize it as an *ordinary* theory, namely in terms of a holomorphic prepotential. We assume higher derivatives to be irrelevant. The non-commutativity and the UV/IR mixing should enter via the prepotential. Note that an IR description of the $U(N)$ theory in terms of a set of non-commutative $U(1)$'s does not make sense since the non-commutative $U(1)$ theory is asymptotically free. Thus, we assume an ordinary (commutative) description in the IR.

The summary of our results is the following: we find that the two-derivative effective action flows in the IR to the commutative one and there is no UV/IR mixing. Namely, the $U(N)$ theory is described in terms of a free $U(1)$ (the center) and a set of $N - 1$ additional $U(1)$'s which are described by the ordinary Seiberg-Witten curve. In particular, we do not find in the IR any coupling between the center $U(1)$ and the other $U(1)$'s. We should emphasize, however, that this is probably due to the two-derivatives approximation that we use. At higher derivatives we do expect a coupling of the center $U(1)$ and the rest of the $U(1)$'s.

The organization of the manuscript is as follows: section 2 is devoted to field theory analysis whereas section 3 is devoted to string theory analysis. In section 2.1 we review the perturbative behavior of the model. In section 2.2 we describe the IR behavior of the

² Shortly after the first version of our paper was published in hep-th, the paper [13] appeared. The authors arrive at the same conclusion that we have put forward, namely that the $SU(N)$ part is unchanged compared to the commutative case and that the $U(1)$ part is free in the IR at least at the two derivative level to which we have restricted our analysis.

$U(1)$ theory. Section 2.3 is devoted to a discussion about the expected behavior of the $U(N)$ model. In section 2.4 we present its solution. Section 3.1 and 3.2 are devoted to the description of the solution from the type IIA and M-theory points of view. It is used to support our findings in field theory.

2 Field Theory Analysis

2.1 Perturbative Dynamics

Non-commutative gauge theories exhibit an interesting perturbative behavior. First, it is important to note that, within the framework of [2] only certain gauge groups and representations can be consistently described over the non-commutative space. In particular we cannot discuss $SU(N)$ theories, only $U(N)$. The reason is that the $SU(N)$ theory is not gauge invariant under the generalized non-commutative gauge transformation since it contains anti-commutators of generators [14, 15]; see however also [16].

The planar graphs of the non-commutative theories are exactly the same as the planar graphs of ordinary theory, apart from overall global phases [17]. On the other hand, non-planar graphs are believed to be UV finite [18]. However, these graphs lead to a new kind of infra-red divergences [3]. Since the source of these new type of divergences is at high momenta, there is an interesting UV/IR mixing. For supersymmetric gauge theories the only new IR divergences are logarithmic, in contrast to the quadratic IR divergences in the non-supersymmetric theory [4]. At one loop, these effects appear only in amplitudes which involve external legs in the center of the gauge group [19]. For $\mathcal{N} = 4$ theory these effects cancel [4] and in the infrared the theory converges to the ordinary one [5, 6].

It is important to note that even though the non-commutative field theories have been shown to have a ‘nicer’ UV behavior than the corresponding commutative theories, a proof of renormalizability has not been given yet. However, since these theories can be realized as a vacuum of string theory via brane configurations, they should be consistent at the quantum level, even in the field theory limit. (The simplest way of realizing non-commutative $\mathcal{N} = 2$ theories in type IIA, is to consider type IIB with D3 branes on a Z_2 orbifold singularity with a constant NS-NS two-form background and then T-dualize to have D4 branes suspended between NS5 branes compactified on a circle [20]).

We assume that the $\mathcal{N} = 2$ model is renormalizable (one loop evidence for the renormalizability of the model is given in [21]). If the model is renormalizable, the structure of divergences should be the same as of the planar commutative theory. Namely, the same counter-terms are needed to regularize the theory and the beta-function consists of only the one loop contribution which is [19]

$$\beta = N_f - 2N \tag{2.1}$$

for an $U(N)$ theory with N_f hypermultiplets. In particular the pure $U(1)$ theory is asymptotically free [22, 23]. We will restrict ourselves to theories without matter, though the generalization to the cases when matter is present is straightforward.

2.2 The $\mathcal{N} = 2$ $U(1)$ Non-Commutative SYM

$\mathcal{N} = 2$ non-commutative Yang-Mills can be formulated in superspace similarly to the ordinary theory [24] (see also [25] and [26, 27] for a recent application to the $\mathcal{N} = 4$ case). The $\mathcal{N} = 2$ gauge multiplet contains a complex scalar field in the adjoint representation. In contrast to the commutative theory, the NC theory is not free and there is a potential for the scalar. It is

$$V \propto (\phi \star \bar{\phi} - \bar{\phi} \star \phi)^2, \quad (2.2)$$

where the square is with respect to the star product. As usual, the classical moduli space of vacua is determined by finding physically inequivalent static solutions of $V = 0$.

Assuming that the minima of the potential are unchanged by quantum corrections, they are

$$\phi = a = \text{constant}. \quad (2.3)$$

There is, however, a crucial difference between the commutative and the non-commutative theories. Usually, in the ordinary $SU(N)$ theory there is a moduli space parameterized by vev 's of gauge invariant monomials of the scalar fields. Here, giving a vev breaks the gauge symmetry spontaneously. In the $U(1)$ non-commutative case the transformation

$$\phi \rightarrow \phi + a, \quad (2.4)$$

with constant a is a symmetry of the theory, i.e. there is no moduli space which is parameterized by a . Therefore, not only the vacuum is insensitive to this shift, but all the physics. For example, there are no W -boson masses which depend on a . Note that this symmetry holds also at the quantum level. This happens since both the action and the measure are invariant under the shift (as we shall see in the type IIA picture this corresponds to the fact that we have the freedom to change the location of the D4 branes without changing the physics on these branes).

One may therefore wonder what the theory in the IR is.

The symmetry (2.4) is spontaneously broken in the vacuum. The realization of this breaking is the appearance of a Goldstone boson. We assume here that the Goldstone theorem holds in the non-commutative case [28]. One might wonder if this is indeed true, as the proof of Goldstone theorem assumes locality. However, non-commutativity is a mild non-locality, and in particular, we observe that there is a Goldstone boson in the UV regime. It is the scalar itself. The realization of the gauge theory via type IIA would give us further support that a Goldstone mode exists. By completion of the $\mathcal{N} = 2$ multiplet, we arrive to the conclusion that we should have a massless $\mathcal{N} = 2$ vector multiplet in the IR, as well. What is therefore the form of the low-energy effective action ?

To lowest order in derivatives, namely to quadratic order, the action can be written in terms of a prepotential \mathcal{F}

$$\int d^2\theta \frac{\partial^2 \mathcal{F}(A)}{\partial A^2} W_\alpha W^\alpha. \quad (2.5)$$

Since the shift (2.4) is a symmetry of the theory, the effective coupling cannot depend on a . It means that the prepotential must be of the following form

$$\mathcal{F} = \frac{1}{2} \tau_0 A^2, \quad (2.6)$$

and at the quadratic level the IR theory is free, exactly as the commutative one. At first sight this looks surprising, since the UV theory is asymptotically free. However, note that the action (2.5) describes the dependence on the 'moduli' A at the IR and not the coupling constant as a function of the energy. Namely, though the gauge coupling runs, we cannot read its running from the prepotential. The reason is that the vev of the scalar is *not* related to the energy scales, in contrast to the ordinary case where W bosons exist.

We may recall that the standard way of deriving the one-loop prepotential is to find an effective action which reproduces the $U(1)_{\mathcal{R}}$ anomaly (see e.g. [29]). Supersymmetry requires the divergence of the $U(1)_{\mathcal{R}}$ to be controlled by the one-loop beta function. In the present case the one-loop beta function does not vanish and thus neither does the \mathcal{R} -anomaly (see [30] for a related discussion). Neither of them is smooth in the $\theta \rightarrow 0$ limit. A quadratic prepotential, however, cannot reproduce the $U(1)_{\mathcal{R}}$ anomaly. Nevertheless, we claim that there is no contradiction between the shift symmetry and the existence of the $U(1)_{\mathcal{R}}$ anomaly. Moreover, a naively expected $\log A$ term in the effective action would, as in [2], require additional singularities which signal the appearance of massless states. We find such a scenario unlikely.

2.3 The $U(N)$ case

Now we turn to the case of non-commutative $U(N)$ Yang-Mills theory. At the generic point on the moduli space the theory is broken to $U(1)^N$. The infra-red theory is described by N independent vector multiplets. In order to see this, let us use the following generators to describe the Cartan subalgebra

$$\tilde{T}^{\tilde{i}} = \text{diag}(0, \dots, 0, 1, 0, \dots, 0) \quad (2.7)$$

Accordingly there are N photons $\tilde{A}_{\mu}^{\tilde{i}}$, $\tilde{i} = 1, \dots, N$. It is important to observe that the residual non-commutative gauge symmetry does not mix the photons $\tilde{A}_{\mu}^{\tilde{i}}$. Indeed, under NC gauge transformation, each photon transform into itself in the following way

$$\tilde{A}_{\mu}^{\tilde{i}} \rightarrow \tilde{A}_{\mu}^{\tilde{i}} + \partial_{\mu} \lambda^{\tilde{i}} + \sin(\theta^{\nu\rho} \partial_{\nu}^{(1)} \partial_{\rho}^{(2)}) \tilde{A}_{\mu}^{\tilde{i}} \lambda^{\tilde{i}}, \quad (2.8)$$

where the derivative $\partial^{(1)}$ acts on the photon and the derivative $\partial^{(2)}$ acts on the gauge parameter. The non-commutative parameter has only non-vanishing space-space components, e.g. $\theta^{12} \neq 0$.

The IR theory thus consists of N photons. However, the low-energy action should be parameterized by only $N - 1$ vev 's. The reason is that the vev of the 'center of mass' $U(1)$ is not a modulus. This is as in sect. 2.2. In order to see this, it is better to parameterize the Cartan by the traceless generators

$$T^i = \tilde{T}^i - \tilde{T}^{i+1} = \text{diag}(0, \dots, 0, 1, -1, 0, \dots, 0) \quad (2.9)$$

and by the 'center of mass' generator $T^0 = \mathbb{I}$. The theory is invariant under a shift of the 'center of mass' scalar by a constant.

Thus the low energy effective action should be written in the following way

$$\int d^2\theta \left(\frac{\partial^2 \mathcal{F}(A, A_{\{i\}})}{\partial A^2} W_{\alpha} W^{\alpha} + \sum_{ij} \frac{\partial^2 \mathcal{F}(A, A_{\{i\}})}{\partial A_i \partial A_j} W_{\alpha}^i W^{\alpha j} \right), \quad (2.10)$$

where A stands for A_0 . By the same arguments as in the previous section it is clear that the prepotential should be of the form

$$\mathcal{F} = \frac{1}{2}\tau_0 A^2 + f(A_{\{i\}}) \quad (2.11)$$

Therefore, the center of mass $U(1)$ is decoupled from the rest of the $U(1)$'s. This is precisely the same behavior as in the commutative case. The $U(N)$ commutative theory decouples to a trivial (center) $U(1)$ and $U(1)$'s which are associated with the broken $SU(N)$. The difference is that the non-commutative theory in the UV does not split into two non-interacting pieces. Rather, the non-commutative $U(1)$ interacts with the $SU(N)$ part. Only at low energies such a decoupling occurs.

2.4 The Seiberg-Witten curve

With the understanding (2.11) we can proceed in a similar way as in the commutative case. The generalization of the Seiberg-Witten curve to the $U(N)$ case seems to be [31, 32, 33]

$$y^2 - 2P(z)y + 1 = 0 \quad (2.12)$$

$$P(z) = \prod_{i=1}^N (z - a_i) = z^N - \sum_{i=0}^{N-1} u_i z^{N-i} \quad (2.13)$$

where, in particular, $u_0 = \sum a_i$. In the type IIA picture discussed below, the D4 branes end on the NS5 branes at the positions a_i and u_0 is proportional to their center-of-mass position.

The kinetic term if the effective action can be written as [34]

$$S = \int d^4x K_{i\bar{j}} \partial_\mu u_i \partial^\mu \bar{u}_{\bar{j}}$$

with $i, j = 1, \dots, N$. Where

$$K_{i\bar{j}} = \int_\Sigma d^2z \frac{z^{N-i-1}}{y - P(z)} \frac{\overline{z^{N-j-1}}}{\overline{y - P(z)}} \quad (2.14)$$

Note that $K_{0\bar{0}}$, i.e. the kinetic energy associated with the center-of-mass, diverges. In fact, by going to center-of-mass coordinates via the shift $z \rightarrow z + \frac{1}{N} \sum a_i$ we can eliminate u_0 so that (2.13) effectively only depends on $N-1$ parameters. Thus, the solution at low-energy reduces to the ordinary solution, at the two-derivatives approximation. An indication that this is indeed the case is given by the gravity solution [35]. Note however that the gravity solution is valid only at large N and therefore the issue of the 'center of mass' $U(1)$ cannot be addressed in this approach.

At first look it seems to contradict the UV/IR mixing of the non-commutative theories. There are one-loop non-planar graphs, associated with the $U(1)$, which indicate a mixing of the UV/IR. In fact, the one-loop effective action analysis which takes into account the UV/IR mixing also suggests a flow to a free $U(1)$ theory [11]. We suggest that at very low energies, the $\mathcal{N} = 2$ theory flows to a commutative theory.

3 The picture from string theory

3.1 Type IIA

The realization of non-commutative $\mathcal{N} = 2$ SYM is as in the commutative case, but with an additional constant NS-NS two-form along the 1, 2 components. The brane configuration consists of N coincident $D4$ branes in the 0, 1, 2, 3, 6 directions and two parallel NS5 branes in the 0, 1, 2, 3, 4, 5. The $D4$ branes spans a finite segment of the 6-direction and the NS5 branes are located at the ends of this segment [36].

The type IIA pictures captures the classical theory and the one-loop effects [36]. It does not describe the IR theory, only the UV. Let us see how it describes the $U(N)$ non-commutative theory.

First of all, it describes really $U(N)$ and not $SU(N)$. The reason is the following. Let us assume that the D-branes are separated. In the presence of NS-NS field there is a modified potential. The potential, due to their positions, is

$$\text{tr} (\phi \star \bar{\phi} - \bar{\phi} \star \phi)^2. \quad (3.1)$$

Let us shift the position of the branes by a 4d space-time dependent piece.

$$\phi \rightarrow \phi + a(x). \quad (3.2)$$

Without the NS-NS background (or, equivalently, without the star product), this shift does not affect the potential, only the kinetic term of the $U(1)$. However, the modified potential is not invariant under this shift. It means that the $U(1)$ field is not decoupled. On the other hand, when a is constant, it is still a symmetry. One can always shift the D-brane positions by a constant. That means that the vev is not a modulus. This shows that the symmetry (2.4) holds at the quantum level, at least perturbatively. The fixing of the position of the branes breaks this symmetry and the result is a Goldstone boson, exactly the same as in the ordinary case.

The bending of the NS5 brane is not affected by the NS-NS 4d background. It describes the dependence of the gauge coupling on the vev of the scalars. This is why the one-loop beta function of the non-commutative theory is the same as the commutative one. The case of a single $D4$ brane (the $U(1)$ theory) is exceptional. In this case, even in the commutative theory, the NS5 branes bend. However, it does not imply a running of the gauge coupling as a function of the energy since a vev (a separation of $D4$ branes) is needed to relate the bending to the “real” beta function.

3.2 M-theory

In the M-theory picture the brane configuration becomes a single $M5$. The configuration is non-compact and the non-compactness carries the additional $U(1)$, which is located at ‘infinity’. Thus the low energy theory (even in the commutative case) is in fact $U(1)^N$ and not $U(1)^{N-1}$. The point is that the additional $U(1)$ is not dynamical. No W -bosons are associated with it. Therefore, the conclusion in the commutative case is that this additional $U(1)$ must be free.

In order to realize non-commutativity, a constant 3-form background should be added [37]. However, the 3-form background does not change the geometrical picture. The $U(1)$ case is described by a single M5 with zero genus. The $U(1)$ mode that lives on the zero genus M5 is non-normalizable and hence it supports our conclusion that the $U(1)$ theory becomes free in the infrared. Let us turn now to the $U(N)$ theory. The compact part of the M5 is still a genus $g = N - 1$ Riemann surface. So, it seems that also in this background there are only $N - 1$ dynamical photons and the additional $U(1)$ decouples from the dynamics.

We now provide some details. First we briefly summarize the structure of the theory (see for details [38, 36]). The harmonic decomposition of the self-dual two-form on the M5 worldvolume gives rise to $U(1)^g$ gauge fields, while two out of five scalars, which parameterize the position of the curve Σ in the four-dimensional manifold $Q = \mathbf{R}^3 \times S^1$ ($\mathbf{R}^3 = \{x^4, x^5, x^6\}$) should be decomposed in the basis of the deformations of the normal bundle thus giving g complex scalars needed to complete $\mathcal{N} = 2$ vector multiplets. Finally, an extra scalar arises from the "decomposition" of the two-form in terms of the volume form of the finite part of Σ (alternatively seen as a Hodge-dual of the two-form in four dimensional space-time.). This compact mode joins the three remaining scalars on the fivebrane worldvolume to form a "universal" hypermultiplet. As is [39], one can show explicitly by looking at the reduction of susy transformations that this multiplets decouples from the rest of the theory.

Dealing with the non-compactness of Σ as in [36], one expands the field-strength of the two-form in terms of the harmonic one-forms ω_i on Σ as

$$H = \sum_i^g (F_i \wedge \omega_i + *F_i \wedge *\omega_i) \quad (3.3)$$

to find at the linearized level that the dynamics is governed by the intersection matrix $\tau_{ij} = \int_{\Sigma} \omega_i \wedge *\omega_j$. One has to bear in mind that in the full theory H is not closed and is related to a closed three-form in a complicated non-linear fashion. While this nonlinearity greatly complicates the full analysis, it is not very important in the far infrared regime where we are working and where the theory is broken to $U(1)^g$. Most importantly, turning on the bulk C -field does not lead to additional deformations of this sector, and it is precisely this deformations that we are interested in. As we will argue now the only deformation induced by the C -field affects the center of mass $U(1)$, which also unfortunately is the weakest point of the M-theory derivation of SW theory (See e.g. [40]).

The one-form, ω_0 , related to the center of mass $U(1)$, requires special attention. This is the only one-form on Σ that extends to Q (note that ω_0 is fixed on Σ) and hence is orthogonal to ω_i ($\tau_{0i} = 0$). Moreover when extended to the bulk it should coincide with the only one-form on Q , namely the form supported on the M-theory circle. In order to obtain the relevant coupling, with a slight abuse of notation and omitting the pull-back signs, we take $C = B \wedge \omega_0$. The C -field coupling to M5, $\int_{M_6} |H - C|^2$, gives a coupling to the center of mass $U(1)$ while leaving the other $U(1)$ fields unaffected:

$$\int_{M_4} |F_0 - B|^2. \quad (3.4)$$

As mentioned before the coefficient in front of this term is infinite but in the far infrared we can absorb it by a field redefinition ³. Unfortunately this argument only shows that the

³ As already mentioned we are not discussing the matter fields, but at this point it is natural to ask

C -field coupling deforms only the center of mass $U(1)$ and that in the far infrared it does not affect the remaining gauge fields, confirming our field-theoretical findings. This conclusion is in complete agreement with the supergravity analysis [35] which shows that the commutative and the non-commutative moduli spaces coincide at large N . However since the adequate description of the center of mass is notoriously difficult in this picture, it does not shed much light on the nature of the $U(1)$ theory itself, and here we will have to rely solely on field theory arguments.

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what happens in situations corresponding to type IIA limit where we have multiple fivebranes. While the M-theoretic discussion, in particular the assertion that there is only one $U(1)$ field which can be coupled to the NS two-form via (3.4), does not change, at first sight the situation looks much different both from IIA and field theory point of view. However it is not hard to see that even in the case of multiple fivebranes (product gauge groups) there is only *one* Goldstone field and thus only one additional massless $U(1)$. Indeed, mutual motion of the groups of fourbranes in the adjacent areas of the interior of the fivebrane chain affects the masses of the hypermultiplets [36], and thus the only allowed shift now is that of the whole system. Field-theoretically speaking, one can also see that any shift, other than simultaneous shift in all scalars, will change the mass of the bifundamental hypers due to the change in the Yukawa couplings.

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